NON-GREY RADIATION MODELING USING THERMAL DESKTOP/SINDAWORKS
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ABSTRACT

This paper provides an overview of the non-grey radiation modeling capabilities of Cullimore and Ring's Thermal Desktop® Version 4.8 SindaWorks software. The non-grey radiation analysis theory implemented by Sindaworks and the methodology used by the software are outlined. Representative results from a parametric trade study of a radiation shield comprised of a series of v-grooved shaped deployable panels is used to illustrate the capabilities of the SindaWorks non-grey radiation thermal analysis software using emissivities with temperature and wavelength dependency modeled via a Hagen-Rubens relationship.

INTRODUCTION

In order to acquaint the reader with pertinent terminology used herein, we begin with a brief synopsis of thermal radiation heat transfer theory. Planck's Radiation Law states that

$$E(\lambda, T) = \frac{2\pi\, hc^2}{\lambda^5 (e^{hc/\lambda kT} - 1)}$$

where h = Planck's constant, k = Boltzmann's constant, c = speed of light, T = temperature, λ = wavelength, E = emissive power. The emissive flux is the area under a particular curve of constant temperature $E(\lambda, T = const.)$, and is given as $Q_{bb,T} = \sigma T^4$, where σ = 5.6697 $\times 10^{-8}$ W/m^2-K is the Stefan-Boltzmann constant. From the Planck distribution, $\lambda_{max} \propto \frac{1}{T}$. The

method used to compute total emissivity when temperature dependency must be considered is given by the following averaging scheme:

$$\varepsilon(T) = \frac{\sum E_{bb,\lambda,T}\varepsilon_{\lambda,T}\Delta\lambda}{\sum E_{bb,\lambda,T}\Delta\lambda} = \frac{\sum E_{bb,\lambda,T}\varepsilon_{\lambda,T}\Delta\lambda}{\sigma T^4}$$

which is a local averaging technique over finite $\Delta\lambda$ intervals.

THERMAL DESKTOP'S NON-GREY METHODOLOGY

In Thermal Desktop, non-grey radiation problems are handled by breaking the problem up into wavelength bands, such that within each band, the problem becomes grey again. This so-called "banded approach" effectively takes care of the emissivity as a function of temperature changes. The temperature dependent spectral emissivities, $\varepsilon(T)$ are handled iteratively. New radiation matrices (a matrix for each band) must be computed for each new temperature, until convergence to equilibrium is accomplished. Using the banded approach, with one band Thermal Desktop used the total emissivity and radiates using the total emissive power of σT^4. With multiple bands, the problem is sub-divided into sub-bands as follows:

$$0 \rightarrow \lambda_1, \lambda_1 \rightarrow \lambda_2, \lambda_2 \rightarrow \lambda_3, \ldots, \lambda_n \rightarrow \infty$$

Thermal Desktop also assumes that the absorptivity is equal to the band averaged emissivity, since absorptivity approaches emissivity as the spacing between the bands becomes narrower. In each sub-band, each node radiates with the amount of energy in that band, rather than the total of σT^4. The banded energy balance is given as follows:

For one band, the radiated energy is

$$Q_{1\leftrightarrow2} = K_{1\leftrightarrow2}\sigma\left(T_2^4 - T_1^4\right)$$

while for multiple bands

$$Q_{1\leftrightarrow2} = K_{1\leftrightarrow2}^{0\leftrightarrow\lambda_1}\left(F_{band}(0,\lambda_1,T_2)\sigma T_2^4 - F_{band}(0,\lambda_1,T_1)\sigma T_1^4\right)+$$

$$K_{1\leftrightarrow2}^{\lambda_1\leftrightarrow\lambda_2}\left(F_{band}(\lambda_1,\lambda_2,T_2)\sigma T_2^4 - F_{band}(\lambda_1,\lambda_2,T_1)\sigma T_1^4\right)+$$

$$K_{1\leftrightarrow2}^{\lambda_2\leftrightarrow\lambda_3}\left(F_{band}(\lambda_2,\lambda_3,T_2)\sigma T_2^4 - F_{band}(\lambda_2,\lambda_3,T_1)\sigma T_1^4\right)$$

$$+\ldots+$$

$$K_{1\leftrightarrow2}^{\lambda_n\leftrightarrow\infty}\left(F_{band}(\lambda_n,\infty,T_2)\sigma T_2^4 - F_{band}(\lambda_n,\infty,T_1)\sigma T_1^4\right)$$

with the constraint

$$\sum_i F_{band}(\lambda_i,\lambda_{i+1},T) = 1$$

In the above, the quantity $K_{1\leftrightarrow2}^{\lambda_i\leftrightarrow\lambda_{i+1}}$ denotes the RADK (thermal radiation conductor), while the banded fraction radiation function, $F_{band}(\lambda_i,\lambda_{i+1},T)$ gives the fraction of energy radiated between λ_i and λ_{i+1} for a blackbody temperature of T. The banded fraction radiation function is given by Siegel and Howell[1] as follows:

$$F_{band}(\lambda_1,\lambda_2,T) = \frac{\displaystyle\int_{\lambda_1}^{\lambda_2} E_{\lambda b}(\lambda,T)d\lambda}{\displaystyle\int_0^\infty E_{\lambda b}(\lambda,T)d\lambda} = \frac{1}{\sigma T^4}\int_{\lambda_1}^{\lambda_2} E_{\lambda b}(\lambda,T)d\lambda$$

which may be expressed as two integrals as follows

$$F_{band}(\lambda_1,\lambda_2,T) = \frac{1}{\sigma T^4}\left[\int_0^{\lambda_2} E_{\lambda b}(\lambda,T)d\lambda - \int_0^{\lambda_1} E_{\lambda b}(\lambda,T)d\lambda\right]$$

Rearranging the above expression in terms of the product λT affords

$$F_{band}(\lambda_1,\lambda_2,T) = \frac{1}{\sigma}\left[\int_0^{\lambda_2 T}\frac{E_{\lambda b}(\lambda,T)}{T^5}d(\lambda T) - \int_0^{\lambda_1 T}\frac{E_{\lambda b}(\lambda,T)}{T^5}d(\lambda T)\right]$$

$$F_{band}(\lambda_1,\lambda_2,T) = F_{0\rightarrow\lambda}(\lambda_2 T) - F_{0\rightarrow\lambda}(\lambda_1 T)$$

The last expression denotes the emitted energy in the wavelength interval λ_1 to λ_2. The fraction blackbody emissive power function (a.k.a. band fraction function) $F_{0\rightarrow\lambda T}(\lambda T)$ is given by Siegel and Howell[1] and reads:

$$F_{0 \rightarrow \lambda T}(\lambda T) = \frac{2\pi \, C_1}{\sigma \, C_2^4} \int_{\zeta}^{\infty} \frac{\zeta^3}{e^{\zeta} - 1} d\zeta$$

where

$$\zeta = \frac{C_2}{\lambda T}$$

$$C_1 = hc_o^2$$

$$h = \text{Planck's constant} = 6.62 \times 10^{34} \text{ J-s}$$

$$k = \text{Boltzmann's constant} = 1.3806 \times 10^{23} \text{ J/K}$$

$$c_o = \text{Speed of light} = 2.998 \times 10^8 \text{ m/s}$$

Both tabular data and Taylor series formats of $F_{0 \rightarrow \lambda T}(\lambda T)$ are available, as discussed in Siegel and Howell[1].

IMPLEMENTING THERMAL DESKTOP'S SINDAWORKS NON-GREY SOFTWARE

Before using the Thermal Desktop/Sindaworks solver, one must first examine the optical properties and range of temperatures to ascertain whether or not the problem at hand is non-grey or not. Next, one defines the wavelengths and/or temperature dependent properties using one dialog form, under one unique optical property name. Next, the thermal analyst must decide which bands to use for the RADK runs. The software requires that the first band start at zero. As a general rule-of-thumb, the last band should run to at least ten times the peak thermal emission wavelength of the coldest surface being modeled. In our studies a value of 100,000 microns sufficed. Hence, our peak thermal emission wavelength was λ_{max} = 2893/T microns, where T is in units of K. A suggested technique for selecting intermediate band edges, for a situation where there are discrete components at presumably different, largely non-overlaping temperature bands is to use the wavelength of peak thermal emission for the temperature midway between the average temperature of adjacent components. Since Thermal Desktop effectively uses some averaging of the radiative properties within each band, this keeps the properties of each component within the same wavelength band to the greatest degree possible.

Thermal Desktop's RadCAD solver recognizes banded analysis and automatically computes the required optical properties for each band, and automatically computes the RADKs for each band. Sindaworks contains built-in logic to perform the band-fraction functions $F_{0 \rightarrow \lambda T}(\lambda T)$ and handles the appropriate energy balance monitoring. The dynamic link between Sindaworks and RadCAD allows for updating of the temperature dependent emissivities and RADK's.

The task of implementing Thermal Desktop / Sindaworks non-grey radiation capabilities is documented in Figures 1 through 11. Figure 1 depicts the familiar Thermal Desktop Optical Properties database dialog form, which is used to define optical properties. Figure 2 shows the Thermal Desktop User Preferences dialog box, where the user enables the SINDAWORKS solver by clicking on the Thermal Analyzer tab. Figure 3 indicates the Thermal Desktop Optical Properties dialog form where the Wavelength dependency is called out. Here, under the pull down menu of "Use Properties", the user scrolls down to "Wave Length Dependent for RADKs, Basic for Heat Rate Calculations", and the edits the entries on the Wavelength Dependent tabular form. In order to enter a bi-variate look-up table of $\varepsilon = \varepsilon(T, \lambda)$ the user clicks the "Use Vs. Temp" checkbox. This enables a form like that shown in Figure 4, where the data for $\varepsilon = \varepsilon(T, \lambda)$ is entered. The top line are the temperatures [K], the next line the emissivity for a constant wavelength, with temperature varying, 1 micron in this case. The following entry is for the next wavelength, 3 microns in this example, etc. until the maximum wavelength is entered.

To model the temperature dependency of emissivity, the Hagen-Ruben's relationship of Siegel and Howell[1] was utilized. For metals, radiation transfer is primarily a surface phenomenon, with all of the pertinent physics occurring within a few hundred angstroms of the surface. Only very thin foils display any real effects of transparency. The Hagen-Ruben's model used in this study was of the form:

$$r_e(T) = A + B\left[\frac{T - 20}{273}\right]$$

$$\varepsilon(T, \lambda) = C\sqrt{\frac{r_e(T)}{\lambda}} - D\frac{r_e(T)}{\lambda}$$

where $r_e(T)$ is the resistivity of the material in units of Ω-cm, while A, B, C and D are curve fit constants. Typical emissivities given by the Hagen-Ruben's relation generated in an EXCEL spreadsheet are shown in Figure 5.

Once having entered the data into the form of Figure 4, the user may graphically display the $\varepsilon = \varepsilon(T, \lambda)$ data as shown in Figure 6. Figure 6 is a good way to determine which wavelength intervals to use for the non-grey banded analysis. The Thermal Desktop Case Set manager dialog of Figure 7 is used to define the wavelength dependent radiation simulation. Under the Radiation Tasks tab of Figure 8, the user clicks on the Properties button in order to enable the Radiation Analysis Data form of Figure 9. This form is the standard Thermal Desktop dialog box wherein the user controls the number of rays used in the Monte-Carlo simulation. Upon clicking the Advanced Control tab, the dialog box of Figure 10 is displayed. The Wavelength Dependent Properties button of the Radiation Analysis Advanced Control Data dialog form is used to define the sub-bands of wavelength to be used in the banded analysis approach of the non-grey radiation simulation. As shown in Figure 11, this simulation uses the bins defined by $\lambda_1 = 0, \lambda_2 = 5, \lambda_3 = 15,$ etc.

BENCHMARK STUDY

In order to validate the Sindaworks algorithms, several benchmark studies were carried out. Figure 12 shows the set-up for a three parallel shield, bounded by two isothermal walls simulation. This configuration was chosen to benchmark the Sindaworks software due to its simplicity and its relationship to fundamental theory. Figure 13 shows steady state contour plots of temperature. This simple model was exercised in three modes, i) ε = constant, ii) $\varepsilon = \varepsilon(T)$, and iii) $\varepsilon = \varepsilon(T, \lambda)$. Table 1 lists the results obtained by varying the emissivity of the shields. Results from Sindaworks are compared to Siegel and Howell[1]. As shown in Table 1, excellent agreement between the Sindaworks solution and the theoretical result of Siegel and Howell[1] solution is obtained, with the percentage error being less than 1% for all scenarios simulated. The overall impact of non-grey analysis can be seen from the results of Table 2. Table 2 illustrates the results for a five layer spherical shield simulation, where the minimum and maximum temperatures are summarized for three simulations, i) ε = constant, ii) $\varepsilon = \varepsilon(T)$, and iii) $\varepsilon = \varepsilon(T, \lambda)$. As shown in Figure 2, the non-grey analysis predicts a lower maximum temperature (618 K vs. 658 K), and a higher minimum temperature (53 K vs. 32 K). Thus, the non-grey effects are significant.

TYPICAL RESULTS

The authors have been using Sindaworks at NASA JPL in support of an R&D effort which involves the thermal design of a heat shield for a large aperture telescope. The heat shield consists of a series of stacked v-groove radiation shields. The Sindaworks software was used to model the emissivity of each shield as a function of both temperature and wavelength. The problem set up for this R&D effort is shown in Figure 14. Figure 14 shows the sun facing shield, three intermediate shields and the telescope facing shield. The objective of the stacked v-groove shields is to reduce the temperature in stages, from a very hot temperature emanating from the environment, to one on the order of 15 K, nearest to the telescope. Zodiacal effects of added solar back loading and dust emission were included by using Sindaworks User Logic functions written in C++. Sindaworks uses an object oriented structured approach to building the overall thermal model. Parameters used in the zodiacal analysis include the following: effective space node sink = 2.7 K; λ=0,15,30,70,150,400,200,20000 micron wavelength bins, α_{ZODI} = 2E-13, ε_{ZODI}=2.01E-7, Zodiacal IR boundary = 260K, Zodiacal solar boundary = 5785 K . Several parametric simulations were carried out, including variable specularity, transmissivity and absorptivity. For a model size of roughly 3000 nodes, the simulations took an average of 1.5 hours (depending on hour many rays were shot during the Monte Carlo simulation) on a DELL 670 Workstation. Typical contour plots of the temperature of the coldest shield are shown in Figure 15. During the course of our investigation it was discovered that non-grey radiation effects were quite significant. Profound changes in the results were obtained when comparing non-grey to grey baseline runs. Hence, the non-grey capabilities of Thermal Desktop were found to be of great assistance in our modeling endeavors.
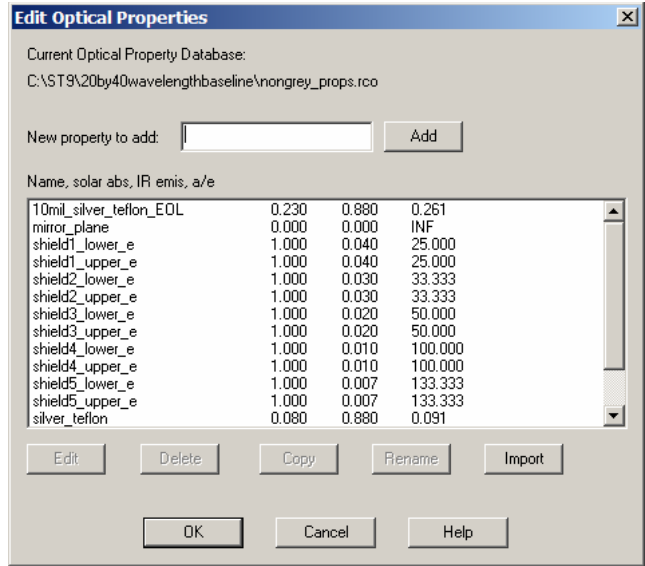
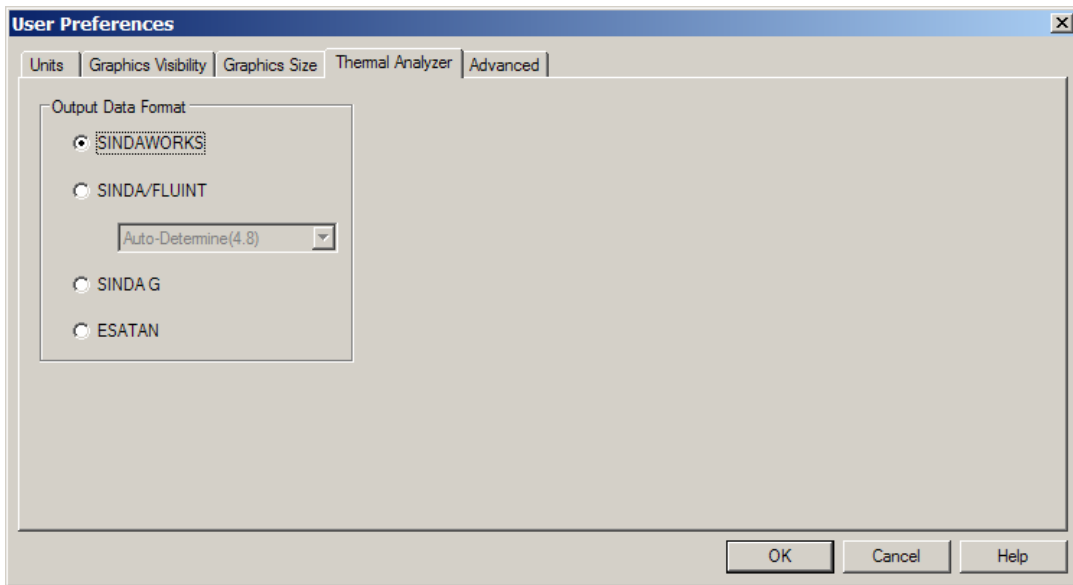**Figure 1: Thermal Desktop Optical Properties Dialog Form**

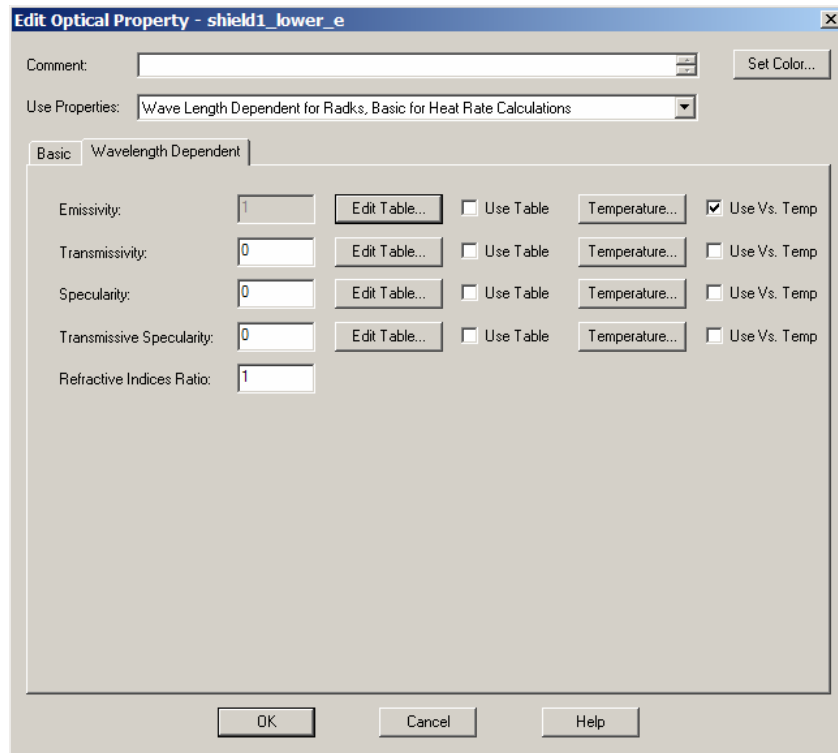

Figure 2: Thermal Desktop User Preferences Dialog Form

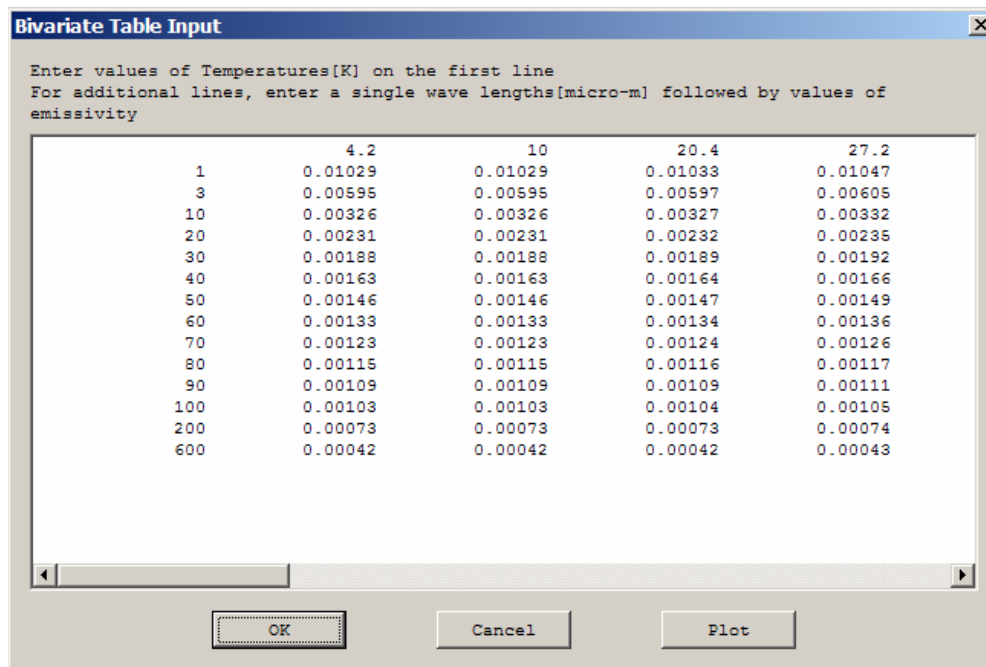**Figure 3: Thermal Desktop Optical Properties Dialog Form**

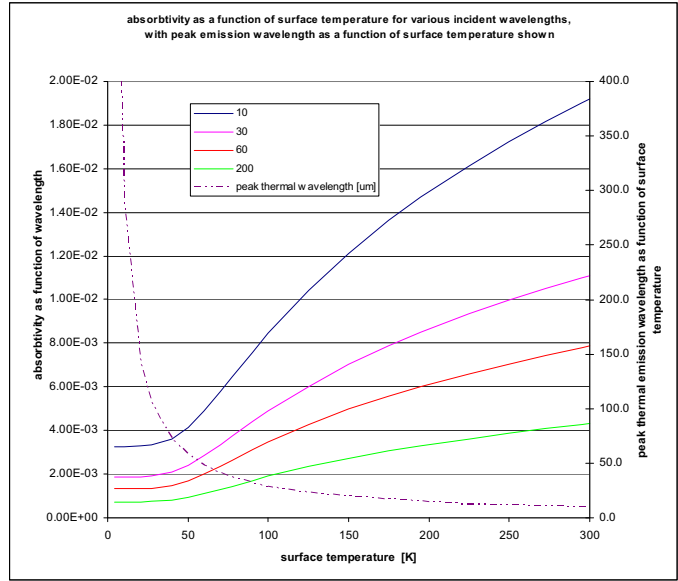**Figure 4: Thermal Desktop Optical Properties Bivariate Array Dialog Form**

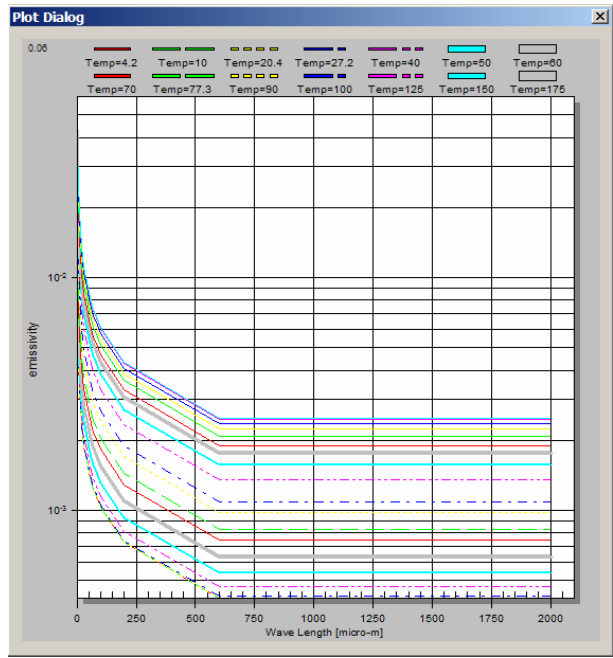**Figure 5: Hagen-Ruben's Approximation for Emissivity as a function of Temperature and Wavelength**

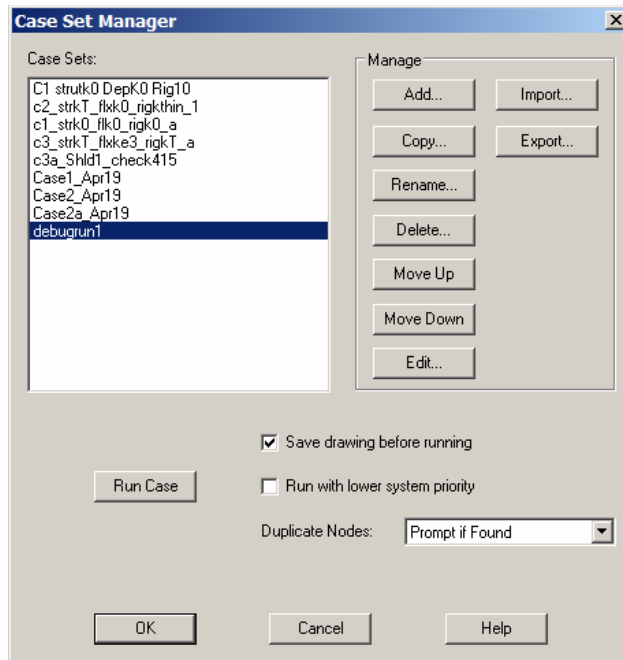**Figure 6: Thermal Desktop Optical Properties Plot Dialog Form**
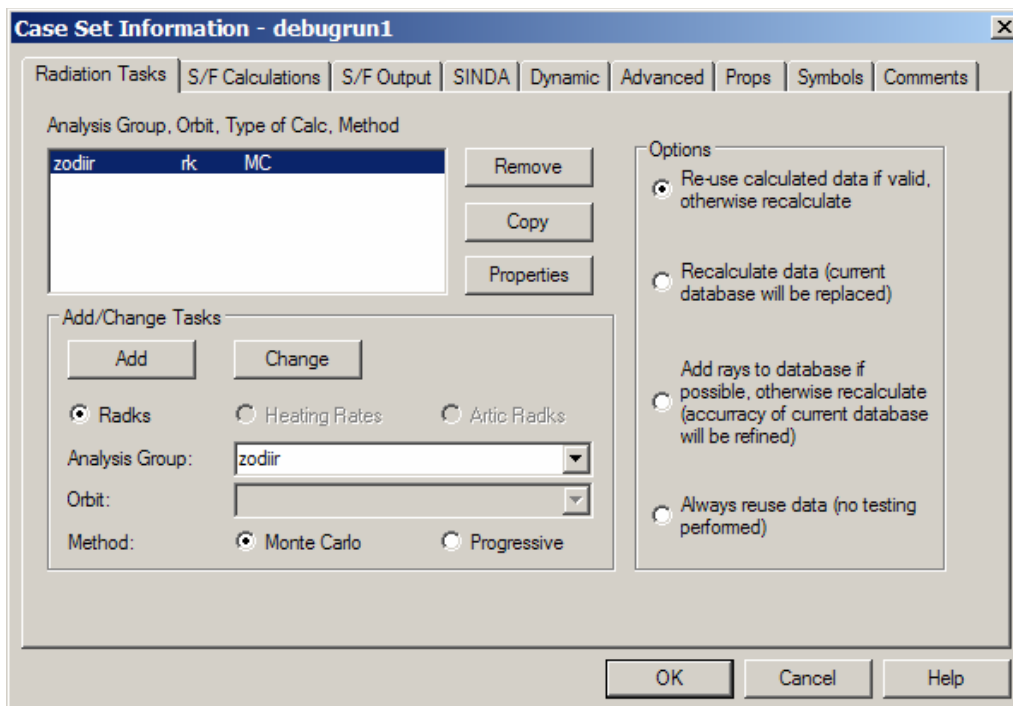
Figure 7: Thermal Desktop Case Set Manager Dialog Form

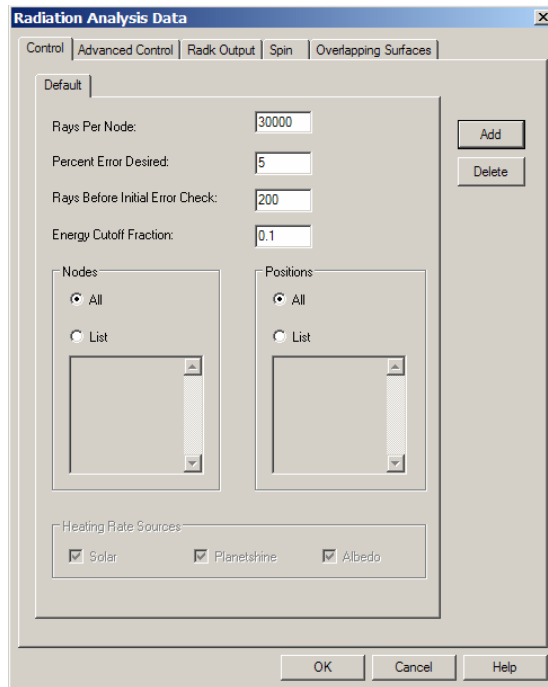**Figure 8: Thermal Desktop Case Set Information Dialog Form**

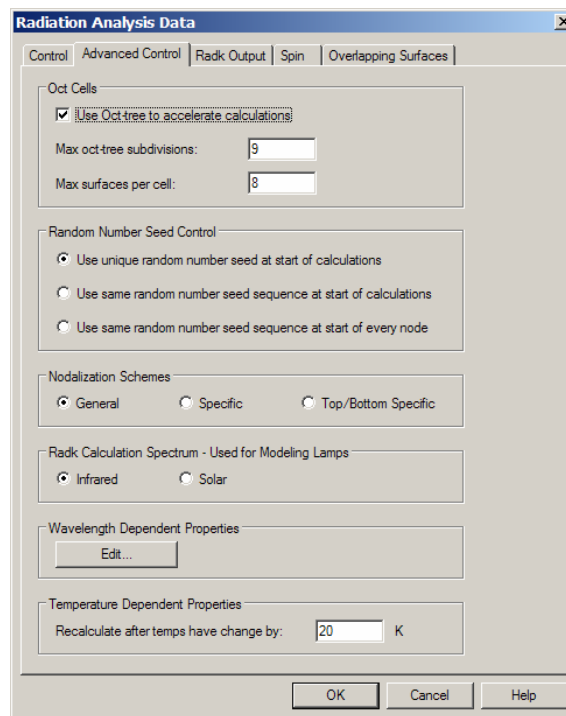**Figure 9: Thermal Desktop Radiation Analysis Data Dialog Form**

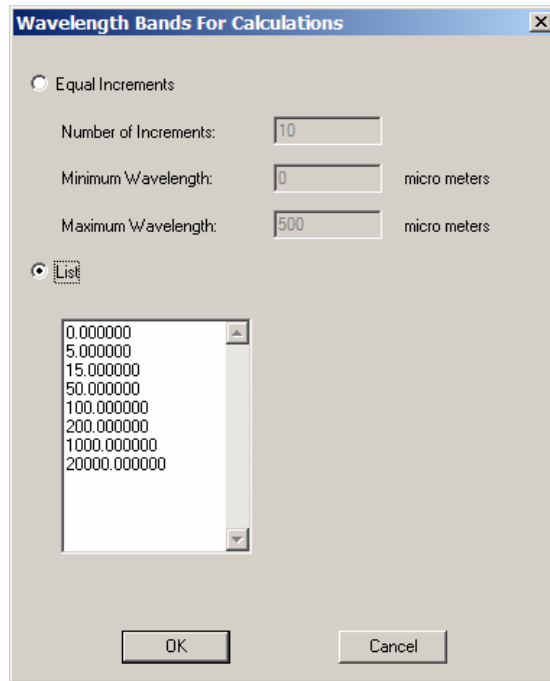**Figure 10: Thermal Desktop Radiation Analysis Advanced Control Data Dialog Form**

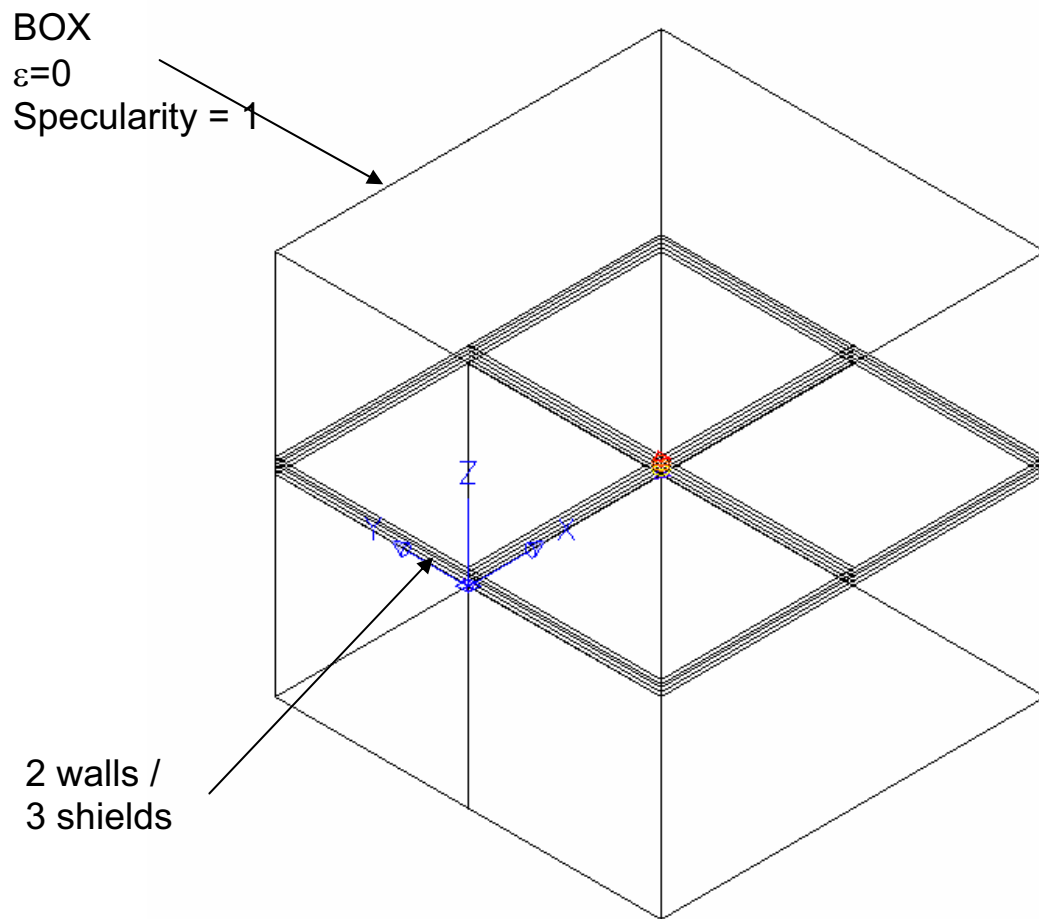**Figure 11: Thermal Desktop Radiation Wavelength Bands Dialog Form**

BOX
ε=0
Specularity = 1

2 walls /
3 shields

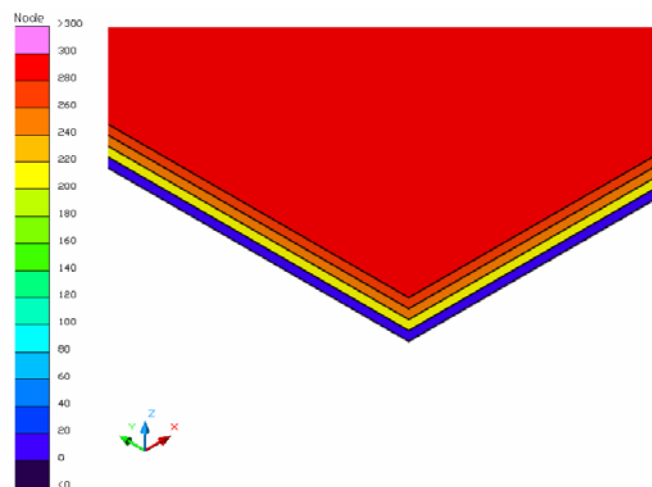**Figure 12: Three Parallel Shields Two Isothermal Walls Benchmark Simulation**
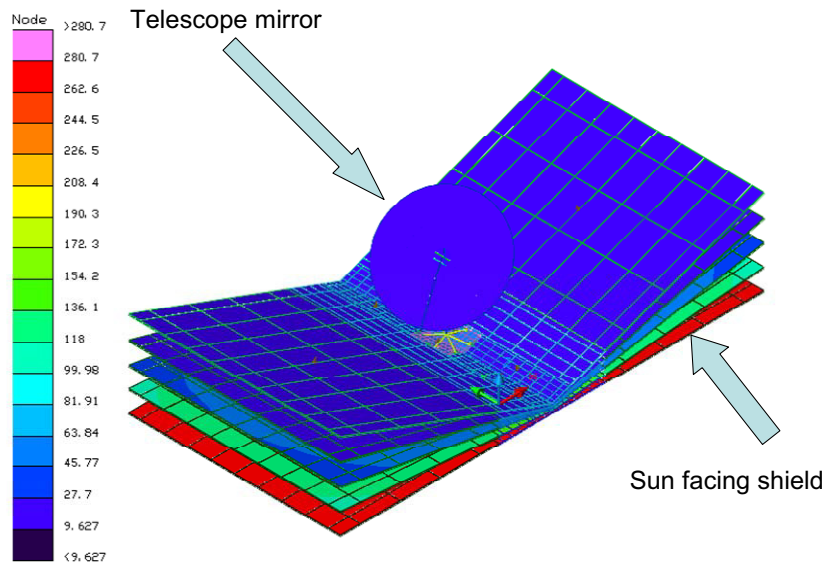
Figure 13: Benchmark Simulation Contour Plots

Figure 14: ST-9 Large Aperture Telescope Model Problem Configuration
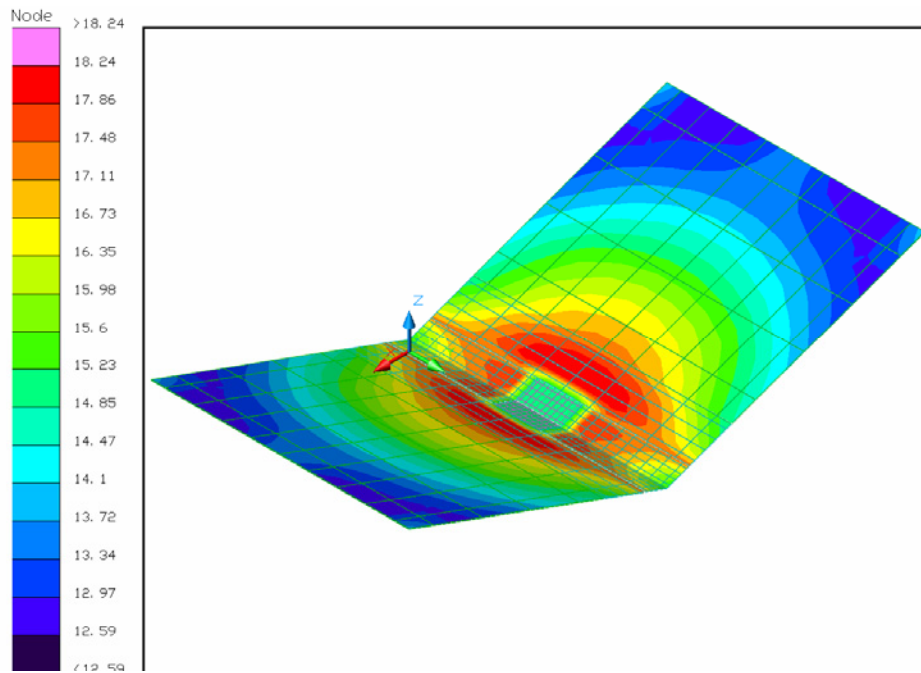

Figure 15: ST-9 Large Aperture Telescope Facing Shield

Table 1. Benchmark Study Results Summary

3 shields, 2 walls Benchmark Radiation Simulation			shield spacing = 0.01 meters					
T wall 1 (K)	300		shield area = 1 m^2					
T wall 2 (K)	0							
σ (W/m^2-K^4)	5.67E-08							
N shields	3							
	SindaWorks	Siegel & Howell Eqn. 9-4		SindaWorks Shield Temperatures (K)				
ε_s	Q (W)	Q(W)	% error	T wall 1	T shd 1	T shld 2	T shld 3	T wall 2
1	114.950	114.818	0.115401	0	212.05	252.24	279.15	300
0.5	38.242	38.273	0.07943	0	212.17	252.28	279.31	300
0.1	6.047	6.043	0.068239	0	211.97	252.16	279.11	300
0.01	0.577	0.577	0.068004	0	211.87	252.18	279.15	300
0.001	0.057	0.057	0.244559	0	212.02	252.15	279.03	300
α_{shield} =	0.23							
Shield/Wall Specularity = 1								
ε_{box} =	0							
Box Specularity =1								

Table 2. Grey versus Non-Grey Sensitivity Study

	Min. Temp (K)	Max. Temp (K)
ε=const.	31.55	658.2
ε=ε(T)	31.77	658.8
ε=$\varepsilon(\lambda,T)$	52.77	617.7

CONCLUSIONS

This paper has introduced the non-grey radiation modeling capabilities of Cullimore and Ring's Thermal Desktop / Sindaworks thermal modeling software. A brief review of the fundamental theory governing non-grey radiative heat transfer used as well as an outline of how the non-grey radiation simulation is modeled in Sindaworks using a "banded" approach has been provided. The effects of using non-grey analysis have been documented by illustrating the effects of non-grey analysis on a simple benchmarking case, as well as on a full scale R&D based simulation study.

ACKNOWLEDGEMENTS

REFERENCES

1. "Thermal Radiation Heat Transfer," 4th Ed. Siegel and Howell, 2002, Taylor and Francis, New York

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